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Supercritical Flow Past a Symmetrical Bicircular Arc Airfoil

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Abstract

Li and Holt (1981) developed a numerical scheme for computing steady supercritical flow about symmetrical airfoils and applied it to an ellipse for zero angle of attack. In this study, an algorithmic description of this new scheme is presented. Application to a symmetrical bicircular arc airfoil is also proposed.

In Li and Holt's scheme, both Telenin's Method and the Method of Lines are used. The flow field before the shock is region 1. For transonic flow, singularity can be avoided by integrating the resulting ordinary differential equations away from the body. Region 2 contains the shock which will be located by shock fitting techniques. The shock divides region 2 into supersonic and subsonic regions and there is no singularity problem in this case. The Method of Lines is used in this region and it is advantageous to integrate the resulting ordinary differential equation along the body for shock fitting.

Coaxial coordinates have to be used for the bicircular arc airfoil so that boundary values on the airfoil body can be taken with one direction of the coaxial coordinates fixed. To avoid taking boundary values at $\pm\infty$ in the coaxial coordinary system, approximate analytical representation of the flow field near the tips of the airfoil is proposed.

1. Introduction

The major problem in transonic flow calculations is the nonlinearity in the governing equations and it causes the partial differential equations to change type within the solution domain, from elliptic in the subsonic region to hyperbolic in the supersonic region. The embedded supersonic region ends with a shock, which causes an additional problem of handling the shock wave.

There are two major categories of finite difference techniques developed to solve the transonic flow problem. One solves the transonic small disturbance equations and the other solves the full potential equations.

In the first category, Murman and Cole (1971) developed the first efficient successive line overrelaxation (SLOR) method for the transonic small disturbance equations. They used a mixed finite difference system. The use of elliptic or hyperbolic difference formulas depends on whether the flow field is subsonic or supersonic. Shock capturing was used to locate the shock. Krupp and Murman (1972) extended the method to lifting airfoils and slender bodies. Later Ballhaus, Jameson and Albert (1978) developed an implicit approximate factorization algorithm for the solution of steady transonic small disturbance equations and found it to have a better rate of convergence than the SLOR algorithm.

In the second category Steger and Lomax (1972) treated the transonic lifting airfoils by solving the full potential equations using SLOR. Jameson (1974) improved it by introducing a rotated differencing scheme in which the direction of upwind differencing is rotated to conform with the local flow direction. The most recent work in this category known to this author was done by Holst and Ballhaus, who solved the full potential equations in conservation form to ensure conservative shock capturing. They later used an arbitrary mesh for the method and obtained good results.

Interpolation (semi-analytical) techniques that have been used in this problem include the Method of Integral Relations, Telenin's Method and the Method of Lines. However, the scheme developed by Li and Holt is the first one that can be generally applied to symmetrical airfoils and yet is able to locate the shock completely.

In the Li and Holt scheme, the steady two-dimensional full potential equations are solved by both Telenin's Method and the Method of Lines. A doublet solution for flow past a closed body is used as the far field boundary condition. The strength of the doublet is a function of both the profile of the airfoil and the flow field. Therefore, it is an unknown and iterations on the strength of the doublet have to be done to obtain the correct boundary values on the airfoil. Jump conditions of the governing equations are applied across the shock wave so that it is perfectly sharp. Correct shock location is obtained by iterations and checks with the boundary conditions downstream of the shock. The iterations mentioned above are two point boundary value problems and can be done efficiently by Powell's method.

Analytical representation of the flow field near the tips of the airfoil can be approximated by the subsonic small disturbance flow. Analytical representation, as described above, is required for the bicircular arc airfoil because one of the coordinates in the coaxial system approaches infinity at the two ends of the airfoil. These will be further elaborated upon in section 3.1.

2. Formulation of the Problem

In this study, a two-dimensional uniform flow past a symmetrical bicircular arc airfoil at zero angle of attack is considered. The embedded supersonic region near the maximum thickness section is as shown in Fig. 1. It is assumed that the flow is both irrotational and isentropic. In fact, it was shown by Li and Holt (1981) that for thin airfoils with subsonic free streams, the shock wave strength is sufficiently small and entropy changes can be neglected.

2.1 Coaxial coordinates

Before proceeding further into formulation, the coaxial coordinates system used in this study should be introduced. It will be easier to understand the coaxial coordinates system by referring to Fig. 2. In Milne-Thomson (1968), it is defined as

$$z = ic \cot \tfrac{1}{2} \zeta \quad , \quad \zeta = \xi + i\eta \tag{2.1}$$

ξ and η, the coaxial coordinates are defined as

$$x = \frac{c \sinh \eta}{\cosh \eta - \cos \xi} \quad , \quad y = \frac{c \sin \xi}{\cosh \eta - \cos \xi} \quad . \tag{2.2}$$

2c is the chord length of the airfoil. When ξ = constant, Eq. (2.2) is a circle whose center is the point $(0, c \cot \xi)$ with radius $c \operatorname{cosec} \xi$. When η = constant, Eq. (2.2) is a circle whose center is the point $(c \coth \eta, 0)$ with radius $c \operatorname{cosech} \eta$. The metric coefficient h is calculated to be

$$h = c(\cosh \eta - \cos \xi)^{-1} \tag{2.3}$$

u and v are velocities in ξ and η directions, respectively.

2.2 Equations of motion

Continuity

$$\frac{\partial}{\partial \xi}(h\rho u) + \frac{\partial}{\partial \eta}(h\rho v) = 0 \quad , \tag{2.4}$$

Irrotationality

$$\frac{\partial}{\partial \xi}(hv) - \frac{\partial}{\partial \eta}(hu) = 0 \quad , \tag{2.5}$$

Bernoulli's equation

$$H + \tfrac{1}{2}q^2 = H_o = \tfrac{1}{2}q_{max}^2 \quad , \tag{2.6}$$

H is the enthalpy per unit mass, q is the flow speed, and q_{max} the maximum steady expansion speed.

By assuming perfect gas with constant specific heat gives

$$H = \frac{\gamma p}{(\gamma - 1)\rho} \tag{2.7}$$

and isentropic flow which further gives

$$p \propto \rho^{\gamma} \tag{2.8}$$

It follows that

$$\frac{H}{H_o} = \frac{p}{p_o}\frac{\rho_o}{\rho} = \left(\frac{\rho}{\rho_o}\right)^{\gamma - 1} \tag{2.9}$$

Therefore, Eq. (2.6) can be written as

$$\frac{\rho}{\rho_o} = \left(1 - \left(\frac{q}{q_{max}}\right)^2\right)^{\frac{1}{\gamma - 1}} \tag{2.10}$$

If we express all variables in dimensionless form by dividing distances by c, velocities by q_{max}, and the density by the stagnant density ρ_o, retaining the same symbols for the non-dimensional variations, Eqs.

$$\frac{\partial}{\partial \xi} (\rho u h) + \frac{\partial}{\partial \eta} (\rho v h) = 0 \quad , \tag{2.11}$$

$$\frac{\partial (vh)}{\partial \xi} - \frac{\partial}{\partial \eta} (uh) = 0 \quad , \tag{2.12}$$

$$\rho = (1 - u^2 - v^2)^{\frac{1}{\gamma - 1}} \quad . \tag{2.13}$$

Equations (2.11), (2.12) and (2.13) are the three equations of motion with the three variables u,v and ρ.

3. Numerical Methods

Telenin's Method as applied to two-dimensional problems uses smooth interpolating polynomials to represent the unknowns in one of the independent variables. The governing partial differential equations are solved in their original form, and thus we avoid the algebra required in the Method of Integral Relations. The symmetry conditions of the present problem suggest the use of Fourier series of the form

$$u(\xi,\eta) = \sum_{i=1}^{N} a_i(\xi)\cos(i-1)\eta \quad , \tag{3.1}$$

$$v(\xi,\eta) = \sum_{i=1}^{N} a_i(\xi)\sin(i-1)\eta \quad , \tag{3.2}$$

where N is the number of rays. In the present problem, the left most and right most rays do not fall on $\eta = -\infty$ and $\eta = +\infty$ due to the special handling of boundary conditions at these two regions. On jth ray

$$u_j = u(\xi,\eta_j) = \sum_{i=1}^{N} a_i(\xi)\cos(i-1)\eta_j \quad . \tag{3.3}$$

The matrix $\{\cos(i-1)\eta_j\}$ can be inverted to obtain a_i,

$$a_i = \sum_{j=1}^{N} A_{ij}u_j \quad , \quad i = 1,\ldots,N \quad , \tag{3.4}$$

where $\{A_{ij}\} = \{\cos(j-1)\eta_i\}^{-1}$.

From Eq. (3.1),

$$\left(\frac{\partial u}{\partial \eta}\right)_\ell = \frac{\partial u(\xi,\eta_\ell)}{\partial \eta} = \sum_{i=1}^{N} a_i(\xi)(1-i)\sin(i-1)\eta_\ell \tag{3.5}$$

Substituting Eq. (3.4) into (3.5) yields

$$\left(\frac{\partial u}{\partial \eta}\right)_\ell = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} A_{ij}u_j\right)(1-i)\sin(i-1)\eta_\ell \tag{3.6}$$

Equation (3.6) can also be written as

$$\left(\frac{\partial u}{\partial \eta}\right)_\ell = \sum_{j=1}^{N} \left(\sum_{i=1}^{N} A_{ij}(1-i)\sin(i-1)\eta_\ell \right) u_j = \sum_{j=1}^{N} F_{\ell j} u_j \quad (3.7)$$

Similarly,

$$\left(\frac{\partial v}{\partial \eta}\right)_\ell = \sum_{j=1}^{N} G_{\ell j} v_j \quad , \quad \ell = 1,2,\ldots,N \quad , \qquad (3.8)$$

where

$$G_{\ell j} = \sum_{i=1}^{N} B_{ij}(i-1)\cos(i-1)\eta_\ell \qquad (3.9)$$

and

$$\{B_{ij}\} = \{\sin(j-1)\eta_j\}^{-1} \quad .$$

Equations (3.7) and (3.8) are used as interpolations of u and v in the η direction; therefore, the resulting ordinary differential equations can be integrated in the ξ direction. The Method of Lines is very similar. Instead of interpolating polynomials, the η derivatives are approximated by three-point or five-point finite difference schemes.

We can obtain the expressions for $\frac{\partial u}{\partial \xi}$ and $\frac{\partial v}{\partial \xi}$ from Eqs. (2.11), (2.12) and (2.13). From Eq. (2.12)

$$h \frac{\partial v}{\partial \xi} + v \frac{\partial h}{\partial \xi} = h \frac{\partial u}{\partial \eta} + u \frac{\partial h}{\partial \eta} \quad . \qquad (3.10)$$

Therefore,

$$\frac{\partial v}{\partial \xi} = \frac{1}{h} \left[h \frac{\partial u}{\partial \eta} + u \frac{\partial h}{\partial \eta} - v \frac{\partial h}{\partial \xi}\right] \qquad (3.11)$$

But

$$\frac{\partial h}{\partial \eta} = -h^2 \sinh \eta \quad , \quad \frac{\partial h}{\partial \xi} = -h^2 \sin \xi \qquad (3.12)$$

and gives

$$\frac{\partial v}{\partial \xi} = \frac{\partial u}{\partial \eta} - uh \sinh \eta + vh \sin \xi \quad . \qquad (3.13)$$

Similarly, from Eqs. (2.11) and (2.13),

$$\frac{\partial u}{\partial \xi} = \frac{P}{Q} \tag{3.15}$$

where

$$P = 2uv \frac{\partial u}{\partial \eta} - uh \sinh \eta + vh \sin \xi$$
$$+ (\gamma-1)(1-u^2-v^2)[uh \sin \xi + vh \sinh \eta - \frac{\partial v}{\partial \eta}]$$
$$+ 2v \, u \frac{\partial u}{\partial \eta} + v \frac{\partial v}{\partial \eta} \quad , \tag{3.15a}$$

$$Q = (\gamma-1)(1-u^2-v^2) - 2u^2 \quad . \tag{3.15b}$$

Therefore, the singular ellipse is obtained from $Q = 0$, or

$$v^2 + \frac{u^2}{q^{*2}} = 1 \quad , \tag{3.16}$$

where

$$q^{*2} = \frac{\gamma-1}{\gamma+1} \quad . \tag{3.17}$$

All points on the ellipse lie outside the sonic circle, except for $v = 0$, $u = q^*$. Therefore, it is apparent that for the supercritical flow as in the present problem, no singularity will be encountered when integrating in the ξ direction, or away from the body. Therefore, for region 1 (see Fig. 4), we must integrate the resulting ODE's from the body.

The expressions for $\frac{\partial u}{\partial \eta}$ and $\frac{\partial v}{\partial \eta}$ can be similarly obtained when we interpolate in the ξ direction and integrate in the η direction. This is applied to region 2 where supersonic and subsonic regions are separated by the shock. In this case, the singular ellipse is

$$u^2 + \frac{v^2}{q^{*2}} = 1 \quad . \tag{3.18}$$

Any critical or supercritical flow has a point on the body with $u=0$ and $v=q^*$, which is a point on the ellipse given by (3.18). That shows that integration in the η direction in region 1 always leads to at least one singularity at sonic points on or near the body.

3.1 Boundary conditions

For the solution represented by Eqs. (2.11), (2.12) and (2.13), we need the boundary values for u and v on the body and at $\eta = -\infty$. But to fit the shock, we also need the boundary conditions at $\eta = +\infty$ so that iterations can correct the position of the shock by checking with values of u and v at $\eta = +\infty$.

On the body, the normal velocity is zero, or

$$u = 0 \quad \text{for } \xi = \xi_o \quad . \tag{3.19}$$

Since the tangential component v is not known, the far field boundary conditions are needed. Murman and Cole (1971) derived an analytical solution for the far field by using the transonic small-disturbance equation

$$[K\phi_x - \tfrac{1}{2}(\gamma+1)\phi_x^2]_x + \phi_{\tilde{y}\tilde{y}} = 0 \tag{3.20}$$

with the variables and parameters defined by

$$\tilde{y} = \delta^{1/3} y \quad , \quad K = (1 - M_\infty^2)/\delta^{2/3} \quad . \tag{3.21}$$

The far field they obtained is that of the usual doublet for a closed body

$$\phi(x,\tilde{y}) \approx \frac{D}{2\pi K^{\frac{1}{2}}} \frac{x}{(x^2 + K\tilde{y}^2)} + \cdots \tag{3.22}$$

where the doublet strength is

$$D = 2 \int_{-1}^{1} F(x)\,dx + \tfrac{1}{2}(\gamma+1)\iint_{-\infty} \{q_x'(x,y)\}^2 dxdy \ . \tag{3.23}$$

The perturbation velocities

$$q_x' = \phi_x \quad , \quad q_y' = \phi_y \ . \tag{3.24}$$

Therefore

$$q_x' = \frac{D}{2\pi K^{\frac{1}{2}}} \frac{(-x^2 + K\tilde{y}^2)}{(x^2 + K\tilde{y}^2)^2} \quad , \quad q_y' = -\frac{DK^{\frac{1}{2}}}{\pi} \frac{xy}{(x^2 + K\tilde{y}^2)^2} \tag{3.25}$$

with

$$\frac{q_x}{U_\infty} = 1 + \delta^{2/3} q_x' + \cdots \quad , \quad \frac{q_y}{U_\infty} = \delta q_y' + \cdots \tag{3.26}$$

The flow velocities expressed in the ξ and η directions are given by

$$u = \frac{d}{dt}(h\xi) = \frac{1}{h}\Big[\frac{-1}{\sinh\eta\,\sin\xi}\,q_x + \frac{1}{\cos\xi\,\cosh\eta - 1}\,q_y\Big] \ , \tag{3.27}$$

$$v = \frac{d}{dt}(h\eta) = \frac{1}{h}\Big[\frac{1}{1 - \cosh\eta\,\cos\xi}\,q_x - \frac{1}{\sin\xi\,\sinh\eta}\,q_y\Big] \ . \tag{3.28}$$

The coordinate η goes to $\pm\infty$ at the two ends of the airfoil, but remains finite except at a small distance very close to the tips. Analytical representation of the flow field in these two regions has to be found so that boundary conditions can be taken at finite values of η. Figure 3 shows the velocity distribution on the ellipse with thickness ratio $\delta = 0.4$ for $M_\infty = 0.65$ (Li and Holt 1981). The velocity distribution near the two ends appears to be symmetrical. Moreover, the velocity in this region is of Mach number less than 0.8. Therefore, subsonic linearized potential equation (Shapiro 1953) can be applied and used as boundary conditions. The basic equation is

$$(1 - M_\infty^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad . \tag{3.29}$$

It can also be written as

$$\frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad . \tag{3.30}$$

where

$$x' = \frac{x}{\sqrt{1 - M_\infty^2}} \quad . \tag{3.31}$$

If we represent the airfoil profile as

$$y = \delta f(x') \quad , \quad \frac{-c}{\sqrt{1 - M_\infty^2}} < x' < \frac{c}{\sqrt{1 - M_\infty^2}} \tag{3.32}$$

and the potential as

$$\phi = Ux' + \delta\phi_1 \quad , \tag{3.33}$$

where δ is the thickness ratio of the airfoil, then the subsonic velocity distribution is known when ϕ_1 is determined.

We can use plane source with strength $\Gamma(\chi)$ so as to satisfy the boundary conditions of the subsonic small-disturbance flow. Hence

$$\phi_1 = \int_{-c'}^{c'} \frac{\Gamma(\chi)}{2\pi} \ln\sqrt{(x'-\chi)^2 + y^2}\, d\chi \qquad (3.34)$$

where

$$c' = \frac{c}{\sqrt{1 - M_\infty^2}} \quad . \qquad (3.35)$$

On the surface of the thin airfoil

$$\frac{\partial\phi}{\partial y} = \delta f'(x') \frac{\partial\phi}{\partial x'} \quad . \qquad (3.36)$$

If we expand $\frac{\partial\phi}{\partial y}$ and $\frac{\partial\phi}{\partial x'}$ about $y = 0$, substituting them into (3.36) and compare the first order (in δ) terms, we get

$$\frac{\partial\phi_1}{\partial y}(x',0) = u_\infty f'(x') \quad . \qquad (3.37)$$

From Eqs. (3.34) and (3.37), and letting $t = \frac{\xi - x'}{y}$

$$\frac{\partial\phi_1}{\partial y} = \frac{1}{2\pi} \int_{-\frac{x'}{y}}^{\frac{L-x'}{y}} \frac{\Gamma(x'+yt)}{1+t^2}\, dt \quad . \qquad (3.38)$$

Letting $y \to 0$,

$$\frac{\partial\phi_1}{\partial y} = \int_{-\infty}^{\infty} \frac{1}{2\pi} \Gamma(x'+yt) \frac{dt}{1+t^2} \qquad (3.39)$$

$$= \tfrac{1}{2}\Gamma(x') \quad . \qquad (3.40)$$

Therefore, we finally obtain

$$\Gamma(x') = 2u_\infty f'(x') \qquad (3.41)$$

and ϕ_1 can thus be determined.

$$q_x = \frac{\partial\phi}{\partial x} \quad , \quad q_y = \frac{\partial\phi}{\partial y} \qquad (3.42)$$

are then used as boundary conditions for the region where η goes to $\pm\infty$.

3.2 Jump conditions

The shock wave on the downstream side of the supersonic flow over the maximum thickness region of the body is handled by shock-fitting in the present problem. The shock wave is modelled by a jump discontinuity in the solution and the jump conditions are satisfied exactly so that the shock wave is perfectly sharp. Instead of the usual Rankine-Hugoniot relations, the jump conditions can be derived from the equations of motion in the present problem. Applying the two-dimensional form of the divergence theorem to Eqs. (2.4) and (2.5), we get

$$\langle \rho uh \rangle (d\eta)_s - \langle \rho vh \rangle (d\xi)_s = 0 \qquad (3.43)$$

and

$$\langle vh \rangle (d\eta)_s + \langle uh \rangle (d\xi)_s = 0 \qquad (3.44)$$

where $\langle\ \rangle$ denote a jump in the quantity across the shock and subscript s denotes an element in the shock surface. Equations (3.43) and (3.44) can also be written as

$$\langle \rho uh \rangle \eta_s' - \langle \rho vh \rangle = 0 \qquad (3.45)$$

and

$$\langle vh \rangle \eta_s' + \langle uh \rangle = 0 \quad , \qquad (3.46)$$

where

$$\eta_s' = \left(\frac{d\eta}{d\xi}\right)_s \quad , \quad \text{the shock wave angle} \quad . \qquad (3.47)$$

The metric coefficient h is continuous and has the same value on both sides of the shock surface and can be dropped from Eqs. (3.45) and (3.46). Hence the final form of the jump conditions is

$$\langle \rho u \rangle \eta_s' - \langle \rho v \rangle = 0 \quad , \tag{3.47}$$

$$\langle v \rangle \eta_s' + \langle u \rangle = 0 \quad . \tag{3.48}$$

3.3 Implementation of the numerical scheme

Telenin's Method and the Method of Lines as applied to elliptic partial differential equations solve a Dirichlet problem as a Cauchy problem. It is inherently unstable with respect to the prescribed data. This phenomenon is known as Hadamard instability. Jones, South and Klunker (1972) encountered Hadamard instability in applying the Method of Lines and found growth in error proportional to $\exp(N\xi)$, where N is the number of rays and ξ the direction of integration. To have sufficient number of rays to represent the variables near the body, we are thus restricted in the direction of integration, ξ. Following Li and Holt (1981), the present problem can be solved in two stages to overcome the difficulty. In the first stage, a very coarse representation of the variables is used, which enables us to integrate the equations away from the body to the far field without instability problems. We will obtain a supercritical shock free flow, which is unstable and not likely to occur in practical situations. The coarse solution provides a fairly good representation of the flow field away from the body where the flow is smooth. The coarse solution at an intermediate value of ξ, say ξ_i, is used as the outer boundary condition. Therefore, a larger number of rays can be used for the refined solution near the body. We have to choose ξ_i so as to avoid having rays of constant ξ with values close to ξ_i to pass through the sonic line that will cause difficulty when integrate through this line. The two regions of integration are shown in Fig. 4.

An algorithmic description of the numerical scheme proposed for the present problem is presented as follows. First, we need to

(a) obtain the simultaneous ordinary differential equations (Eqs. (3.13), (3.15)) by interpolating the variables in one direction.

(b) estimate the tangential velocity on the surface of the airfoil.

(c) obtain an analytical representation of the flow field near $\eta = \pm\infty$ as boundary conditions.

(d) estimate the doublet strength D for the far field doublet solution (Eq. (3.22)).

The computational procedures are then

1. integrate the ode's with large steps in ξ and η. Use Telenin's Method for the coarse solution to obtain a coarse solution. Integration in the second quadrant only is required as the shock free flow is symmetrical.

2. integrate in ξ direction and compare with the far field doublet solution.

(a) correct the estimates for tangential velocities and repeat until the difference between the far field solutions falls within tolerance. It can be done effectively with Powell's Method which will be briefly explained in section 3.3a.

(b) recalculate D with the results obtained in 2(a) and repeat 2 and 2(a) until values of D converge.

3. the coarse solution hence obtained at ξ_i is used as boundary conditions for the refined solution.

4. divide the flow field into two regions (Fig. 4) and refine the mesh.

5. repeat the procedure 2 for the refined solution in region 1.

6. For region 2, the ode's are integrated in the η direction.

7. estimate the location of the shock. The shock wave is normal on the surface. The jump conditions ((3.47), (3.48)) are fully specified when the shock location is known.

8. integrate in the η direction, apply jump conditions across the shock. Check with the downstream boundary conditions.

9. correct the shock location with Powell's Method.

10. Lastly, recalculate doublet strength D with the solution obtained and repeat the whole procedure. The

whole computation is complete when values of D at successive iterations agree to within the prescribed tolerance.

The numerical scheme is briefly summarized by the flow chart in Fig. 5.

3.3a Powell's Method

Powell's Method is essentially that of least squares minimization. A concise description of Powell's method can be found in section 6.9 of Numerical Methods in Fluid Dynamics by Holt [1984]. This method is best explained by example.

In the procedure 2 of the numerical procedure, the difference between the two far field solutions ε_i depends on the initial guess of the tangential velocity, V_j. Then the method minimizes

$$\sum_{i=1}^{N} \varepsilon_i^2 \tag{3.49}$$

with respect to $V_1, V_2, \ldots, V_N$. $\Sigma\varepsilon_i^2$ is minimized by making changes to V_j according to the direction $\delta\vec{V}$ given by

$$\sum_{j=1}^{N} \{ \sum_{k=1}^{N} \frac{\partial \varepsilon_k}{\partial V_i} \frac{\partial \varepsilon_k}{\partial V_j} \} \delta V_j = - \sum_{k=1}^{N} \varepsilon_k \frac{\partial \varepsilon_k}{\partial V_i} \quad i = 1, 2, \ldots, N \tag{3.50}$$

New values of $\vec{V}$ are given by

$$\vec{V} = \vec{V}_{old} + \lambda \delta \vec{V} \quad , \tag{3.51}$$

in which λ is chosen such that $\Sigma\varepsilon_i^2$ is minimized along the direction $\delta\vec{V}$. The required λ can be chosen by evaluating ε_i at different values of λ.

4. Conclusion

The application of the composite numerical scheme developed by Li and Holt (1981) to a bicircular arc airfoil is proposed. As coaxial coordinates have to be used, analytical representation of the flow field near the two tips of the airfoil is required as boundary conditions. The analytical representation can be easily constructed by assuming a subsonic small-disturbance flow at the two ends of the airfoil. An algorithmic description of the numerical scheme is also presented.

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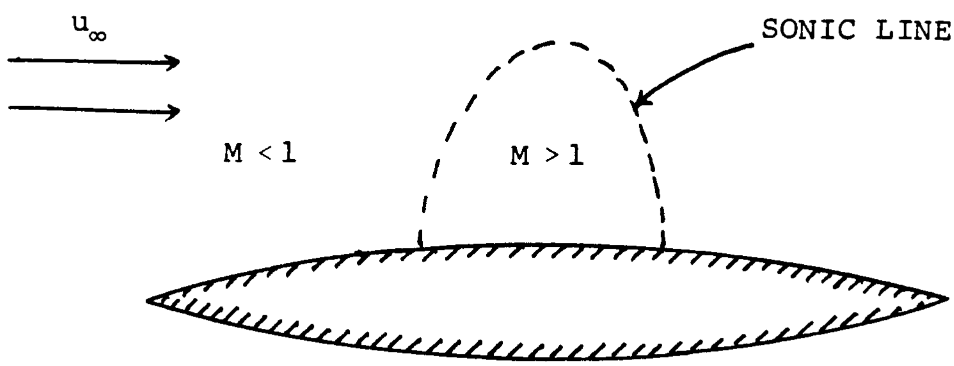


Fig. 1(a) Unstable Supercritical Shock Free Flow Field. Upper Half Plane.

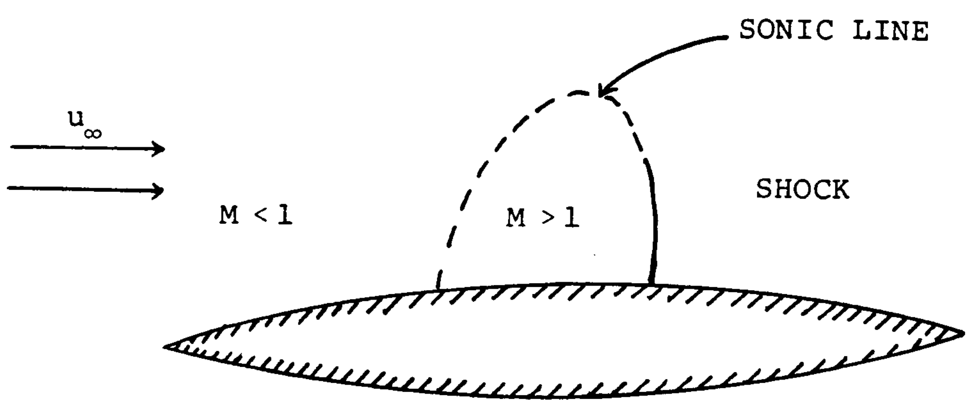
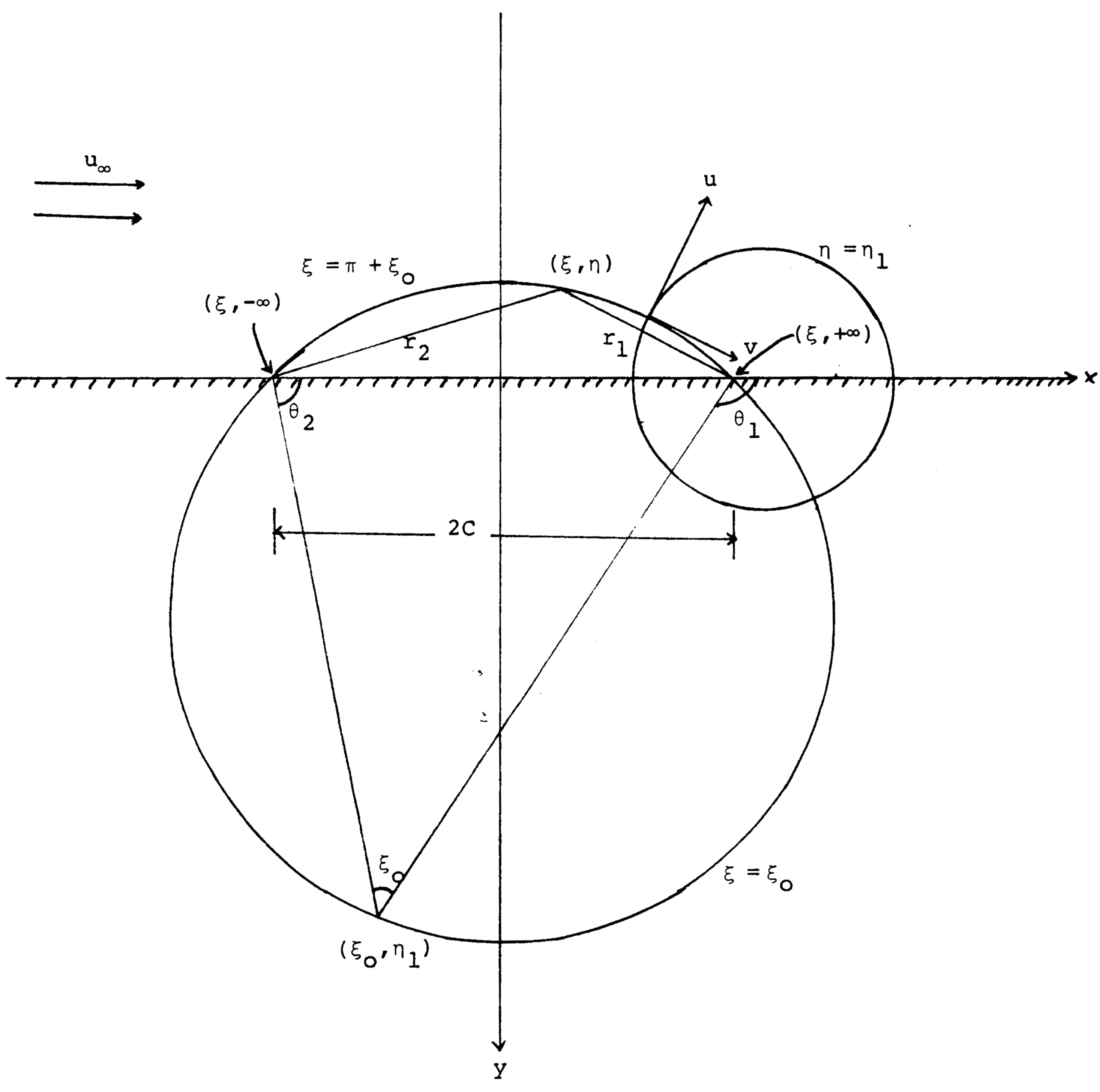


Fig. 1(b) Typical Supercritical Flow Field. Upper Half Plane.

Fig. 2 The construction of coaxial coordinates, $\eta = \ln(r_2/r_1)$, $\xi_o = \theta_1 - \theta_2$. The half-plane with negative y is used.



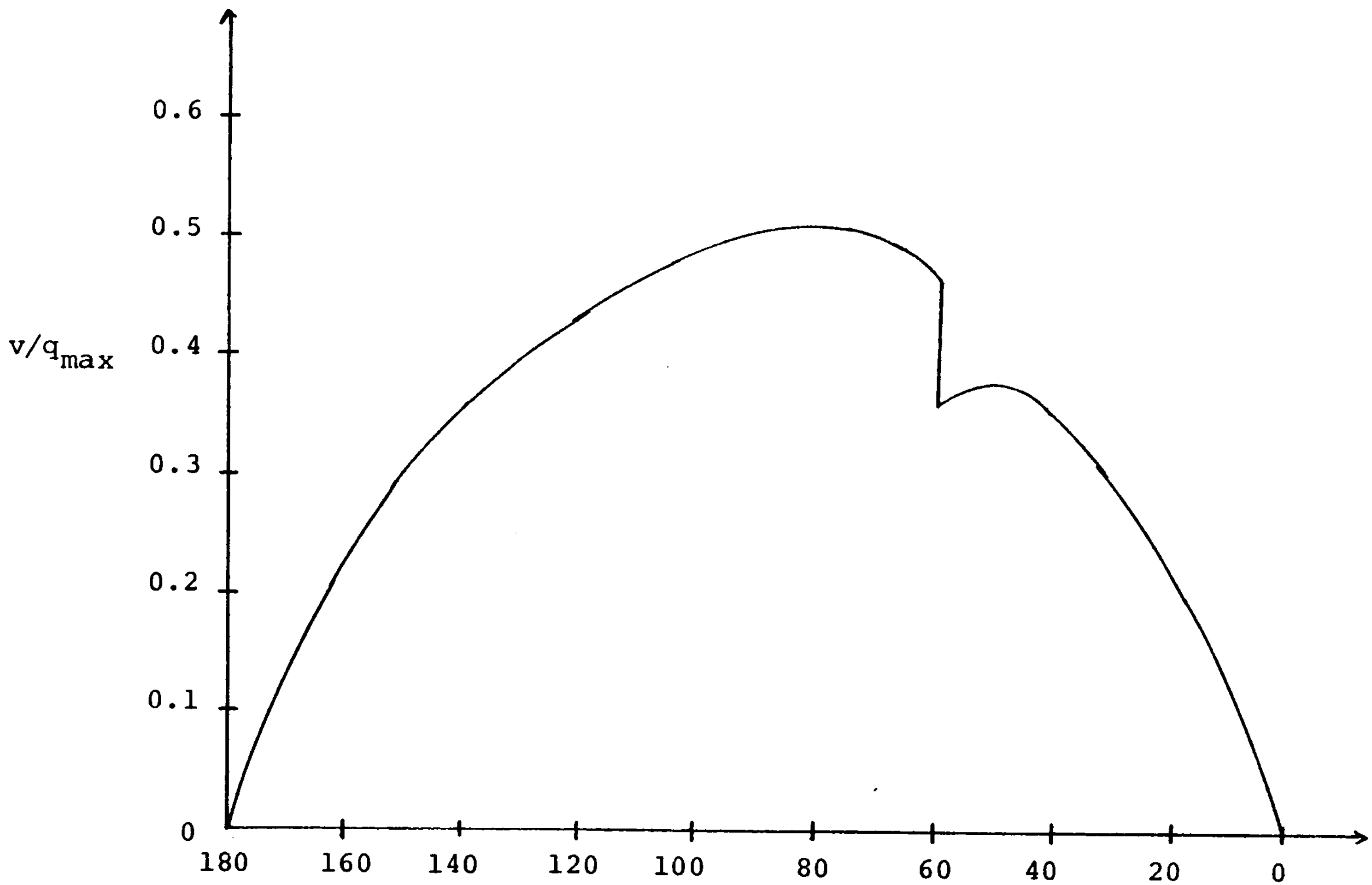


Fig. 3 Surface velocity distribution in supercritical flow in an ellipse, $\delta = 0.4$, $M_\infty = 0.65$ [Li and Holt 1981].

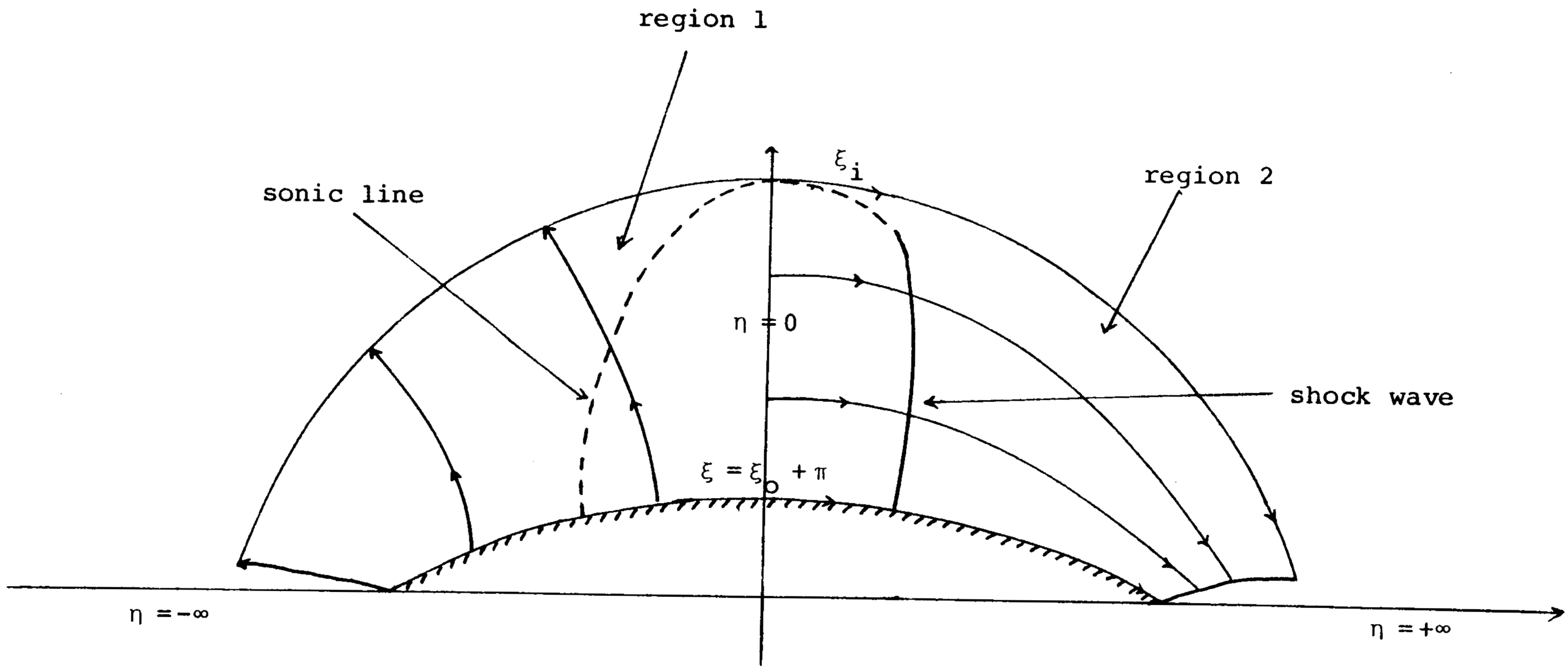


Fig. 4 The two regions of the domain of integration for the refined solution. The directions of integra- are as indicated.

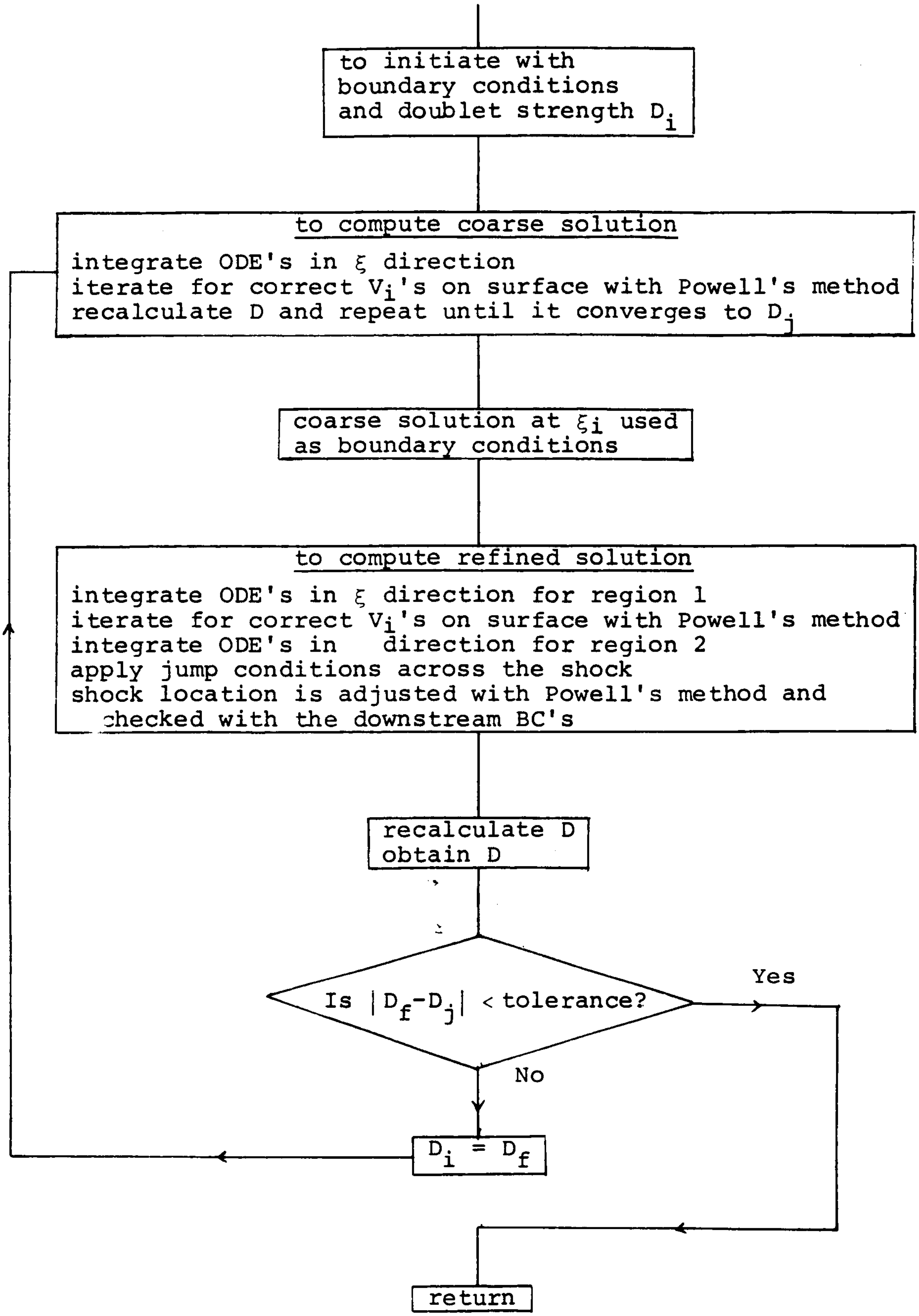


Fig. 5 Flow chart of the numerical scheme